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CALCULATING GAS FLOW IN A HYPERSONIC NOZZLE WITH CONSIDERATION --ETC(U)

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CALCULATING GAS FLOW IN A HYPERSONIC NOZZLE WITH CONSIDERATION OF THE EFFECT OF VISCOSITY (DIRECT PROBLEM)

By

A. P. Byrkin and I. I. Mezhirov

EDITED TRANSLATION

FTD-ID(RS)T-0868-79 10 August 1979

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А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
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М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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CALCULATING GAS FLOW IN A HYPERSONIC NOZZLE WITH CONSIDERATION OF THE EFFECT OF VISCOSITY (DIRECT PROBLEM)

A. P. Byrkin and I. I. Mezhirov

Summary

This report discusses a procedure for the approximate and precise numerical solution of the problem of gas flow in a given hypersonic nozzle with consideration of the effect of viscosity (in the vicinity of the boundary layer). The results of the computer calculation of the flow of helium in a conical nozzle are given.

The following main problems make up the design complex of a hypersonic nozzle:

a) the construction (with consideration of viscosity) of the nozzle contour which provides the assigned flow in an inviscid isentropic core (direct problem). In the vicinity of the boundary layer, the problem is reduced to increasing the through cross sections of the nozzle calculated for inviscid flow by the value corresponding to the depth of displacement of the boundary layer δ*;

b) the calculation of gas flow in a nozzle with a given configuration at a given value of the Reynolds number and temperature of the nozzle wall (direct problem). The solution of this problem is more complex, since the dimensions of the isentropic core and the values of the Mach numbers in it are determined by the interaction of the inviscid flow with the boundary layer, whose parameters, in turn, depend on the characteristics of the inviscid flow.

Among the studies concerned with solving the direct problem for a nozzle, we will point out reports [1] and [2], which describe a method of calculating gas flow in a thin hypersonic conical nozzle. The integral relationship of the pulses is used to calculate the boundary layer, the velocity profile in the boundary layer is assumed to be linear, and the calculations are made for a heat-insulated nozzle wall and a Prandtl number of one. A linear law of the

dependence of the azimuthal velocity component on the polar angle is used in the inviscid core, which makes it possible to consider the nonuniformity of the Mach numbers induced by the boundary layer in the transverse cross section of the nozzle.

The method of successive approximations is sometimes used to solve the direct problem at moderate supersonic velocities in the nozzle, whereupon the condition corresponding to flow of an inviscid gas in the nozzle is used as the zero approximation for calculating the boundary layer. The method of successive approximations is also used for calculating external flows, when the boundary layer interacts with an inviscid flow, including at hypersonic velocities (e.g., see [3]). The process of successive approximations usually converges.

However, one should bear in mind that with a thick boundary layer in the internal problem, the errors in determining the thickness of the boundary layer at hypersonic velocities cause large deviations of the values of the gas-dynamic parameters from the actual parameters due to the limitation of the flow by the walls, which may cause the iteration process to be divergent.

This is illustrated in Fig. 1, which shows the depth of displacement of the boundary layer of the zero approximation δ_0^*

calculated for the conditions of flow of an inviscid gas (helium) in a hypersonic nozzle with an apex half-angle of 6°, a ratio of the throat radius to the exit radius of 0.0181 (the Mach number of the fictitious flow in the absence of a boundary layer on the nozzle walls M_ϕ = 36.5), Re_{0L} = 848•10⁶ and 84.8•10⁶ and with a heat-insulated wall. Here $Re_{0L}=\frac{\rho_0 W_{max} L}{\mu_0}$, ρ_0 : ρ_0 and μ_0 are the density and dynamic viscosity of the isentropically braked gas, respectively; W_{max} is the maximum gas velocity; and L is the length of the nozzle. In Fig. 1 and below,
$\bar{r}_w=\frac{r_w}{r_*}$, $\bar{r}_s=\frac{r_s}{r_*}$, $\bar{x}=\frac{x}{r_*}$, r_w is the current radius of the nozzle cross section, r_s is the radius of the isentropic core, $(r_s=r_w-\delta^*)$, x is the distance from the throat along the nozzle axis, and r_* is the radius of the nozzle throat.

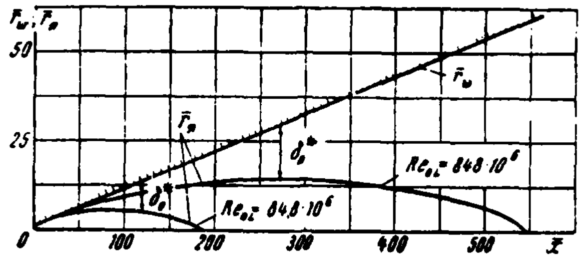

Fig. 1.

It is evident from Fig. 1 that in this case, the calculation leads to an absurd result - the value of δ_0^* inside the nozzle turns out to be equal to the nozzle radius. It is clear that the iteration process will not converge at much smaller values of δ_0^*, either.

Below we will describe an effective method of the approximate and precise numerical solution of the direct problem for a hypersonic nozzle based on certain properties of a laminar boundary layer established by solving the inverse problem for a number of nozzles.

1. Figure 2 shows dependences of the dimensionless depth of displacement of the laminar boundary layer $\frac{\delta^*}{x}\sqrt{\overline{\mathrm{Re}}_{0x}}$ on the number M_1 for a given isentropic contour of the nozzle, calculated by A. P.

Byrkin and Yu. N. Pavlovskiy for a number of profiled axisymmetrical nozzles with a heat-insulating wall. The curves which correspond to values of the Mach number of M = 14.9, 12.9 and 12.2 characterize flow in nozzles with a bend in the generatrix, and the rest - flow in nozzles with a conical section.

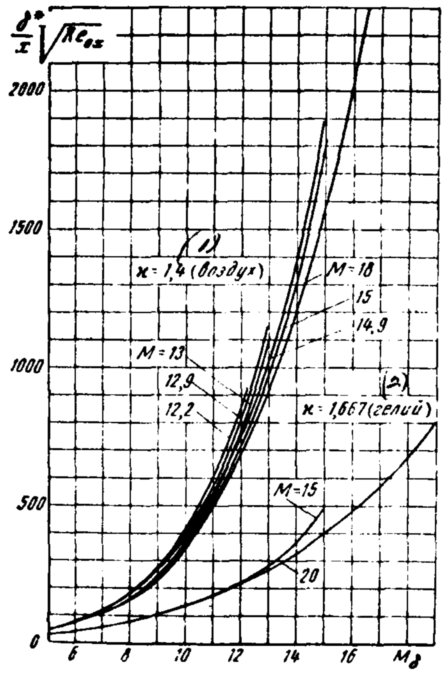

Fig. 2. KEY: (1) air. (2) helium.

The curves were obtained by the numerical integration of the boundary layer equations by A. A. Dorodnitsyn's method of integral relationships (in the third and fifth approximations) [4] and by the method of finite differences [5]. The calculations were conducted for air (adiabatic index $\varkappa$ = 1.4 and a Prandtl number of Pr = 0.75, the dependence of the viscosity coefficient on temperature was taken from the Sutherland formula, T_0 = 1000°K), and for helium ($\varkappa$ = 1.667, Pr = 0.68, the exponent in the viscosity law n = 0.647). The effect

of the transverse curvature of the nozzle contour on the boundary layer characteristics was not considered in the calculations, since in practice it can be disregarded with an error on the order of 1-2o/o at ordinary Re numbers. This is indicated by the results of numerical calculations [6], as well as data of special calculations made by the authors.

It is evident from Fig. 2. that with a given working gas, the dependences obtained for the eight different nozzles differ little from each other. With an error of around ±10o/o, we can consider the value of $\frac{\delta^*}{x}\sqrt{Re_{0x}}$ in hypersonic axisymmetrical nozzles with a heat-insulated wall to be a function of only the Mach number on the edge of the boundary layer, and that it does not depend on the nature of the distribution of the Mach numbers over the length of the nozzle. We will point out that the existence of this universal dependence follows from the laws of similarity established by Yu. L. Zhilin for gas flow in thin affine-like hypersonic nozzles [7].

Figure 3 shows the dependence $\frac{\delta^*}{\delta_1^*}=k(\bar{T}_w)$, plotted for all eight nozzles. Here δ_1* is the depth of displacement at a wall temperature of T_{wr}, which corresponds to the case of heat insulation; δ* is the depth of displacement at a wall temperature of T_w; $\bar{T}_w=\frac{T_w}{T_{wr}}$ is the temperature factor. It is evident from Fig. 3 that the value of δ*/δ_1* is essentially a function of only the temperature factor $\bar{T}_w$.

and it depends weakly on M_δ and the longitudinal pressure gradient in the nozzle, as well as on the physical characteristics of the working gas, at sufficiently large Mach numbers.

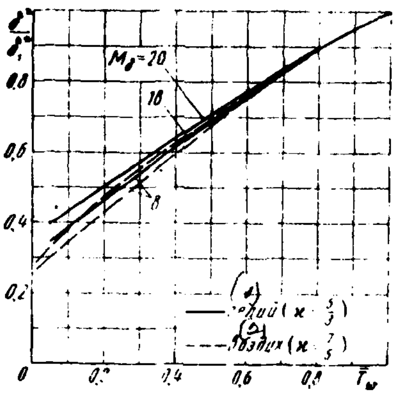

Fig. 3. KEY: (1) Helium. (2) Air.

It follows from the data given that in the first approximation, the depth of displacement on the nozzle wall can be determined from the formula

$$\delta^* = \frac{x}{\sqrt{\mathrm{Re}_{0x}}} f(\mathrm{M}_\delta)\, k(\bar{T}_w), \qquad (1)$$

where $f(\mathrm{M}_\delta)=\frac{\delta_1^*}{x}\sqrt{\mathrm{Re}_{0x}}$, $k(\bar{T}_w)=\frac{\delta^*}{\delta_1^*}$ are universal functions of the Mach number and the temperature factor $\bar{T}_w$, determined for air and helium by the curves in Figures 2 and 3.

2. The data in the preceding section make it possible to obtain

a simple approximate solution to the problem of the gas flow in a nozzle with a fixed configuration during univariate flow in an isentropic core and a laminar boundary layer on the nozzle walls. Flow which is nearly univariate occurs in conical hypersonic nozzles with small opening angles (10-15°). The calculation of flow in the core in the univariate approximation is often also sufficient for profiled nozzles operating in off-design conditions, since it gives us an idea of the deviations of the Mach numbers from the calculated values.

We will write the equation of the flow rate for gas flow in a core, assuming, like in the calculations discussed earlier, that there is no boundary layer in the nozzle throat:

$$F_{я}\, q(M_{я}) = F_{*}. \qquad (2)$$

Here $F_{я}$ and F_{*} are the area of the isentropic core and the nozzle throat, respectively; $M_{я}$ is the Mach number in the core; and q(M) is the derived flow rate:

$$q(M) = \frac{\left(\frac{\varkappa+1}{2}\right)^{\frac{\varkappa+1}{2(\varkappa-1)}} M}{\left(1+\frac{\varkappa-1}{2}M^2\right)^{\frac{\varkappa+1}{2(\varkappa-1)}}}.$$

We have

$$\sqrt{q(M_{я})}\left(\bar{r}_{w} - \frac{\delta^*}{r_{w}}\right) = 1, \qquad (3)$$

whence, using (1), we finally obtain

$$\sqrt{q(M_я)}\left[r_w(x) + \frac{\bar{x}}{Re_*} f(M_я)\, k(T_w)\right] = 1, \qquad (4)$$

where $Re_* = \frac{\rho_0 W_{max} r_*}{\mu_0}$.

Equation (4) can be easily solved graphically for $M_я$ by using Figures 2 and 3 for a nozzle of a given shape at the assigned values of the temperature factor T_w and Re_*. The area of the isentropic core is calculated from the values of $M_я(x)$ using expression (2).

It follows from formula (4) that the number $M_я$ will be constant if all of the abscissas of the points of contour $\bar{x}$ vary in proportion to Re* as parameter Re* changes, while the ordinates r_w corresponding to them remain unchanged. This law of similarity follows strictly from the equation of motion of a viscous gas when the static pressure is constant in the channel cross section (see [8]).

We will point out that in the common case of a conical nozzle, when $\bar{r}_w = a\bar{x} + b$, where a and b are constants, we can obtain the

dependence $\bar{x}(M_a)$ in explicit form. Here equation (4) is reduced to a quadratic equation in $\sqrt{\bar{x}}$, and we obtain:

$$\bar{x}=\left\{\frac{f(M_a)\cdot k(\bar{T}_w)}{2a\sqrt{Re_*}}+\sqrt{\frac{f^2(M_a)\cdot k^2(\bar{T}_w)}{4a^2 Re_*}+\frac{1}{a}\left[\frac{b}{\sqrt{q(M_a)}}-1\right]}\right\}^2$$

For a turbulent or transitional boundary layer, the experimental data on the depth of displacement can also be generalized by a formula of the form

$$\delta^*=\frac{x}{Re_{0x}^{\gamma}}\varphi(M_a,\bar{T}_w) \qquad (5)$$

(e.g., see [9]). The exponent γ obtained is equal to 0.2-0.3. When expressions (5) and (3) are used, we obtain the following equation for determining the Mach numbers in the isentropic core of a nozzle of a given shape:

$$\sqrt{q(M_a)}\left[\bar{r}_w(\bar{x})-\frac{\bar{x}^{1-\gamma}}{Re_*^{\gamma}}\varphi(M_a,\bar{T}_w)\right]=1 \qquad (6)$$

It follows from formula (6) that, like above, the number M_a in the nozzle does not vary if the parameters $\bar{x}\,Re_*^{\frac{1}{1-\gamma}}$ and $\bar{r}_w$, remain constant; one dependence between these parameters defines a whole family of nozzles at different values of Re_*.

We can get an idea of the precision of this method by comparing the results obtained using it with the data of the precise numerical

calculation for a profiled nozzle (Figures 4 and 5 - the solid curves show the results of the precise calculation, and the broken ones - approximate). The contour of the nozzle wall is formed by solving the inverse problem by adding the depth of displacement to the contour calculated without considering viscosity (this perfect contour $r_*(x)$ is the same in Figures 4 and 5). The nozzle is designed to obtain a uniform flow with M = 13 in the characteristic exit rhombus. The wall contour $r_w(x)$ in Fig. 4 corresponds to the case of a heat-insulated wall (T_w =1), and in Fig. 5 - to the value of the temperature factor T_w = 0.25. Re_* = 0.49•10⁶. The figures show the distributions of the Mach number on an inviscid contour obtained by the precise numerical calculation. They also show the curves of $r_*(x)$ and $M_*(x)$, obtained by solving equation (4) at given r_w, Re_*, T_w. It is evident that the approximate dependences differ little from the precise ones over the entire length of the nozzle, although the depth of displacement of the boundary layer at the nozzle exit even exceeds the radius of the inviscid core.

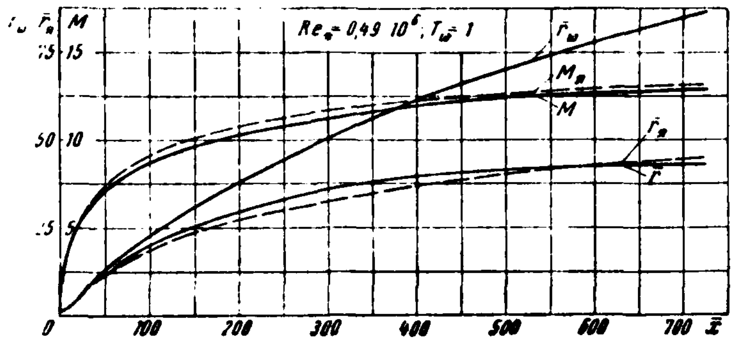

Fig. 4.

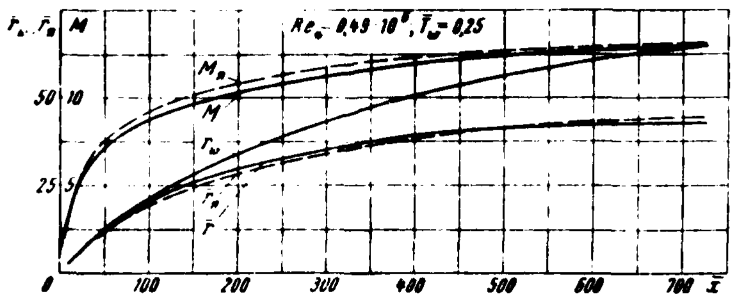

Fig. 5.

3. We can expect the application of the method of successive approximations to the numerical solution of the direct problem of gas flow in a hypersonic nozzle to be successful if the zero

approximation results in a flow pattern close to the true one. As the preceding section shows, we can use the data obtained by solving the approximate equation (4) as the zero approximation.

The results of numerical computer calculations of the flow of helium in a conical nozzle, the main characteristics of which are given at the beginning of the article, at Re_{0l} 848•10⁶, 530•10⁶, 84.8•10⁶ and two temperature conditions on the wall - $T_w = T_{w'}$ and $T_w = T_0$ (T_0 is the stagnation temperature of the gas) are given below.

The boundary layer was calculated by the method of generalized integral relationships in the third approximation.

Axisymmetrical gas flow in an inviscid isentropic core whose contour was determined by the equation

$$r_e(x) = r_w(x) - \delta^*(x), \qquad (7)$$

was calculated by the method of characteristics using a specially written program for solving the direct problem. There were at least 125 points on each characteristic. It was assumed that there is no boundary layer in the nozzle throat. In order to avoid the detailed calculation of the transonic section of the nozzle, it was assumed that radial gas flow at M = 1.01 occurs immediately after the throat.

The Mach number on the boundary of the isentropic core, obtained from formulae (5) and (2) in the zero approximation, was used to calculate the depth of displacement of the boundary layer of the first approximation. Then new values of the radius of the inviscid core, etc., were determined until the approximation yielded virtually the same results with identical satisfaction of relationship (7).

Since the iterations fluctuate around an unknown limiting value at supersonic velocities in an inviscid core (this follows from the main gas-dynamic relationships - the area of the channel increases as the M number increases - and the fact that the value of $\frac{\delta^*}{x}\sqrt{Re_{0x}}$ is an increasing function of the Mach number), "damping" was used to improve convergence: the radius of the inviscid core in the i-th approximation, which is used to calculate the flow of the inviscid gas, was calculated from the formula

$$r_{i\,\text{расч}} = r_{i-1} + l(r_i - r_{i-1}),$$

where the damping coefficient l was considered to be equal to 0.5-0.25.

The process always turned out to be convergent, and the number of approximations required did not exceed four. The data

corresponding to a heat-insulated wall were used as the zero approximation for the numerical solution of the problem with the boundary condition $T_w = T_0$. Two approximations were necessary in this case.

Figure 6 shows the distribution of Mach numbers over the radius of the nozzle exit section obtained by these calculations. It is evident that at Re_{0L} =848•10⁶ and 530•10⁶, the Mach numbers in the inviscid core markedly decrease with distance from the nozzle axis. When Re_{0L} = 84.8•10⁶, the flow in the core is close to unidimensional.

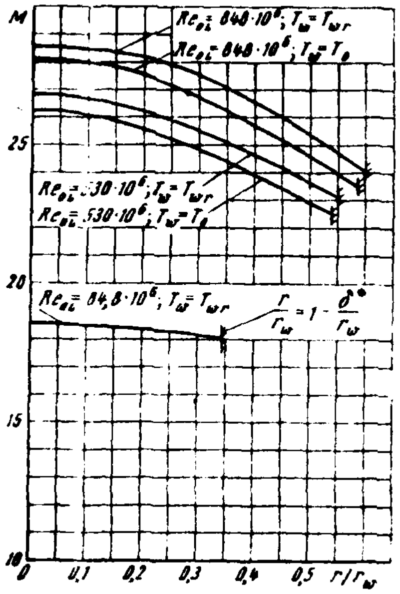

Fig. 6.

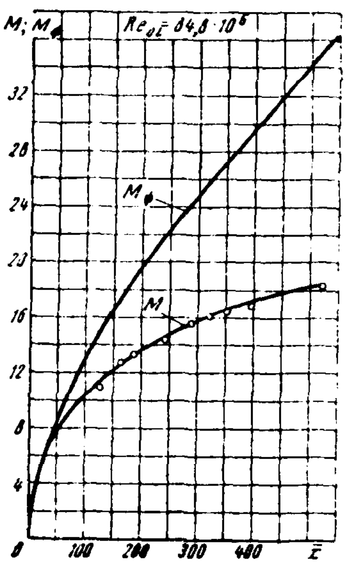

Fig. 7.

The calculated and experimental distributions of Mach numbers on the nozzle axis at Re_{0i} = $84.8 \cdot 10^6$ are compared in Fig. 7 (the experimental study was conducted by V. Ya. Bezmenov and I. I. Mezhirov)[1].

Footnote: [1]The Knudsen number calculated for the nozzle radius did

not exceed $(1-2)\cdot10^{-2}$ in the experiments, which indicates the validity of the comparison of the experimental and calculated data. End footnote.

This figure also shows the curve $M_p(x)$, which corresponds to flow of an inviscid gas in a conical nozzle. The agreement of the calculated and experimental data is satisfactory. The effect of viscosity on the flow in the nozzle is characterized by the difference in the actual values of the Mach numbers from the values of M_p, corresponding to radial flow of an inviscid gas.

Figure 8 shows the velocity profile and the stagnation temperature profile in the boundary layer in the nozzle exit section for $Re_{0L} = 530\cdot10^6$ and a heat-insulated wall (the variable η,

$$\eta = \frac{u_\delta}{u_{max}} \frac{p}{p_{0\delta}} r_a \sqrt{Re_*} \frac{y}{r_*},$$

is proportional to the distance from the wall y; u_δ and $T_{0\delta}$ are the values of the velocity and stagnation temperature on the outer edge of the boundary layer, respectively). The graph shows the value of η_{δ^*}, corresponding to the depth of displacement. It is evident that the depth of displacement differs insignificantly from the thickness of the boundary layer.

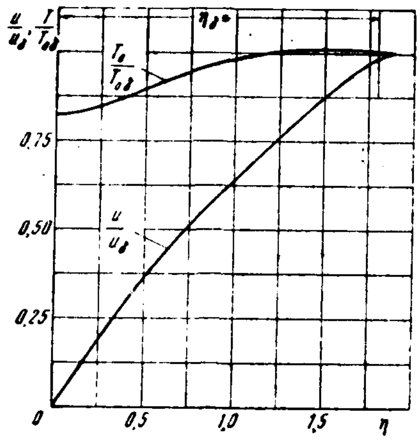

Fig. 8.

In conclusion, we will point out that with consideration of viscosity, the procedure given here for the numerical calculation of gas flow in a nozzle with a given shape is valid for an arbitrary nozzle with a sufficiently smooth contour and an arbitrary gas at arbitrary boundary conditions on the wall. It is only limited by the requirement of the validity of using the boundary layer equation (i.e., for example, the condition of the absence of shock waves interacting with the boundary layer in the nozzle, the condition of sufficient smallness of the effects of the rarefaction of the gas).

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Bibliography

1. V. P. Agafonov. Interaction of a boundary layer with a hypersonic flow in a conical nozzle. "Bull. of the AS USSR. Mechanics," 1965, No. 5.

2. V. P. Agafonov. Asymptotic nature of hypersonic flow in a conical nozzle. "Bull. of the AS USSR. MZhG", 1967, No. 5.

3. W. Mann, R. Brady. Hypersonic viscid-inviscid interaction solutions for perfect gas and equilibrium real gas boundary layer flow. "J. of Astronautical Sci.", Vol. 10, No. 1, 1963.

4. Yu. N. Pavlovskiy. Numerical calculation of laminar boundary layer in a compressible gas. ZhVM i MF, 1962, No. 5.

5. A. A. Byrkin, V. V. Shchennikov. Numerical method of calculating a laminar boundary layer. ZhVM if MF, 1970, No. 1.

6. V. V. Mikhaylov. Method of calculating supersonic nozzles with consideration of the effect of viscosity. "Bull. of the AS USSR.

MZhG.", 1969, No. 1.

7. Yu. L. Zhilin. Laws of similarity for escape of a gas into a thin hypersonic nozzle. "Engineering journal", 1963, No. 4.

8. A. P. Byrkin, I. I. Mezhirov. Calculation of flow of a viscous gas in a channel. "Bull. of the AS USSR. MZhG.", 1967, No. 6.

9. A. F. Burke, K. D. Bird. Use of conical and profiled nozzles in hypersonic installations. In the coll. "Modern techniques of aerodynamic studies at hypersonic velocities," M., "Mashinostroyeniye", 1965.

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